Lesson 8-2: Similar Polygons

Close but no cigar…

Remember when we were exploring all the ways you can prove triangles congruent? One of the options we looked at was AAA (angle-angle-angle). We quickly determined that it wasn't sufficient for congruence. Today we are going to revisit AAA.

Similarity

If we say that two polygons are **similar**, we mean the following two things:

1. Corresponding angles are congruent (this is the AAA part).
2. Corresponding sides are proportional (this is where yesterday's lesson comes in).

The symbol for similarity is the tilde character (~). Thus we'll write *ABCD* ~ *EFGH*.

If you notice, being able to determine corresponding parts is essential for both parts. A quick review: if we say that *ABCD* ~ *EFGH* then to determine side correspondence:

$$ABCD \sim EFGH \quad \text{or } \overline{AB} \leftrightarrow \overline{EF}, \overline{BC} \leftrightarrow \overline{FG}, \overline{CD} \leftrightarrow \overline{GH}, \overline{AD} \leftrightarrow \overline{EH}$$

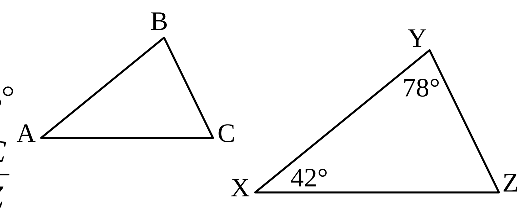
When determining corresponding angles, make sure you use 3 letter naming if there is more than one angle possible at the vertex.

Example

1. $\Delta ABC \sim \Delta XYZ$ … complete each statement:

   a. $m\angle B = ?$ $\angle B$ corresponds to $\angle Y$ so $m\angle B = 78°$

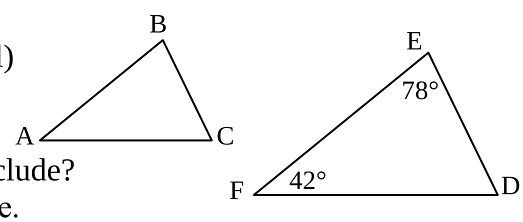
   b. $\frac{BC}{YZ} = \frac{?}{XZ}$ $\overline{XZ}$ corresponds to $\overline{AC}$ so $\frac{BC}{YZ} = \frac{AC}{XZ}$



Similarity ratio

The 2nd requirement for similarity (corresponding sides are proportional) means that the ratios of the lengths of all corresponding sides are equal (proportional). We call this a **similarity ratio**. For example, if $\Delta ABC \sim \Delta FED$

then $\frac{AB}{FE} = \frac{BC}{ED} = \frac{AC}{FD}$ (corr. sides are proportional)

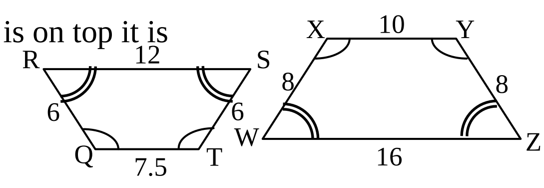


Now, here's a question for you: if two triangles are similar with a similarity ratio of 1:1, what could you conclude? Think about this and I'll answer it at the end of the lecture.

Example

2. **Problem #8, pg 425:** Are the polygons similar? If so, give the similarity statement and similarity ratio. If not, explain.

   If you flip *QRST* over (top-bottom) so *QT* is on top it is easier to see. Also note since both are isosceles trapezoids you could flip *QRST* side-side so that *QR* is on the left or right.



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Correspondence:

$\angle Q \cong \angle X$, $\angle T \cong \angle Y$ *and* $\angle S \cong \angle Z$, $\angle R \cong \angle W$ **(corresponding angles $\cong$)**

and $\overline{QR}$ & $\overline{XW}$, $\overline{ST}$ & $\overline{ZY}$, $\overline{QT}$ & $\overline{XY}$, $\overline{RS}$ & $\overline{WZ}$

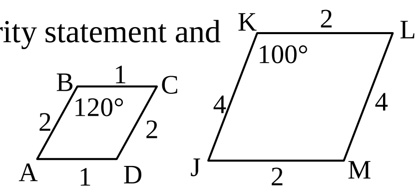
Similarity ratio: $\frac{QR}{XW} = \frac{ST}{ZY} = \frac{6}{8} = \frac{3}{4}$ or 3:4

$$\frac{QT}{XY} = \frac{7.5}{10} = \frac{7\frac{1}{2}}{10} = \frac{\frac{15}{2}}{\frac{10}{1}} = \frac{15}{2} \bullet \frac{1}{10} = \frac{15}{20} = \frac{3}{4} \text{ or } 3:4$$

$\frac{RS}{WZ} = \frac{12}{16} = \frac{3}{4}$ or 3:4 **(corresponding sides are proportional)**

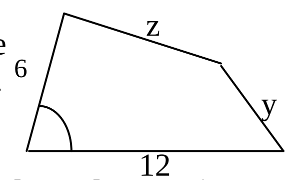
Similar: $QRST \sim XWZY$; $\frac{3}{4}$ or 3:4

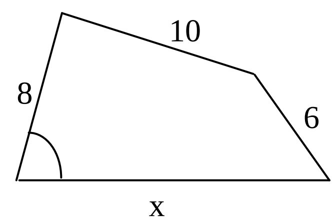
3. Are the parallelograms similar? If so, give the similarity statement and similarity ratio. If not, explain.



No; while corresponding sides are proportional (½), corresponding angles are not congruent.

4. **Problem #15, pg 426:** The polygons are similar. Find the value of each variable.





Match corresponding sides in ratios going from the smaller polygon to the larger. **<u>Be very careful to keep the order consistent!</u>**

$\frac{8}{6} = \frac{x}{12} = \frac{6}{y} = \frac{10}{z}$ (Note where the variables are)

Now solve for each variable:

$\frac{x}{12} = \frac{8}{6}; x = \frac{12 \bullet 8}{6} = 16$ & $\frac{6}{y} = \frac{8}{6}; \frac{y}{6} = \frac{6}{8}; y = \frac{6 \bullet 6}{8} = 4.5$ & $\frac{10}{z} = \frac{8}{6}; \frac{z}{10} = \frac{6}{8}; z = \frac{10 \bullet 6}{8} = 7.5$

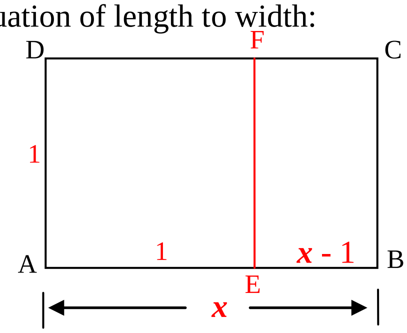
So $x = 16$; $y = 4.5$; $z = 7.5$

The golden ratio

At the start of Book VI of Euclid's Elements is a geometric pattern that is interesting because it represents a geometric ratio that is often found in architecture. This pattern is called the **<u>golden ratio</u>** because most find it very pleasing to the eye. It is approximated by the ratio 1.618:1 (length:width). The golden ratio is seen in the **<u>golden rectangle</u>** which is a rectangle whose length and width are in the golden ratio.

You will often see the golden ratio expressed as a ratio equation of length to width:

$$\frac{l}{w} = \frac{1.618}{1} \text{ or } \frac{l}{w} = \frac{w}{l-w} \text{ or } \frac{l}{w} = \frac{l+w}{l}$$



To see how these equations and the magic ratio of 1.618:1 are found, consider a rectangle of width (height) 1 and length *x*. If you divide this rectangle into a square and smaller rectangle, you will have the following diagram:

If *BCFE* ~ *ABCD* then $\overline{AB}\ corr.\overline{BC}\ \&\ \overline{BC}\ corr.\overline{CF}$
(Length) *(Width)*

(If this looks weird, work through the corresponding side relationships from the similarity statement.)

Thus we have:

$\frac{AB}{BC}=\frac{BC}{CF}$ (corr. sides of ~ polys are prop.)

$\frac{x}{1}=\frac{1}{x-1}$ (subst. - note that this is the $\frac{l}{w}=\frac{w}{l-w}$ equation from above)

$x^2-x=1$ (cross-product prop.)

$x^2-x-1=0$ (subtr. prop.)

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$ (quadratic formula, *a* = 1, *b* = -1, *c* = -1)

$x=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}=\frac{1+\sqrt{5}}{2}$ (only care about positive solution/length)

$x=1.618033989\approx1.618$

Golden ratio example

The length and width of a rectangular tabletop are in the golden ratio. The shorter side is 40 *in*. Find the length of the longer side.

Let the width *w* = 40 and the length be *l*. Form a golden ratio proportion:

$\frac{l}{w}=\frac{1.618}{1}=\frac{l}{40}$ and solve for *l*. $\frac{l}{40}=\frac{1.618}{1}; l=40\bullet1.618=64.72\approx65in$

Similarity ratio of 1:1

So, the question from earlier in the lesson: if two triangles are similar with a similarity ratio of 1:1, what could you conclude? If the similarity ratio is 1:1 that means the lengths of corresponding sides are equal. Equal length means congruence. Thus, we'd have two triangles for which all corresponding sides are congruent. SSS allows us to determine triangle congruence.

Homework Assignment

p. 425 #1-16, 21-28, 31-39, 53-66